Astronomy 271
Spring 2012

Homework \#6
Due Tuesday, April 10 in class

For full credit you must write your solutions neatly and include all work. Do not forget the units.

Universe $9^{\text {th }}$ edition:
Chapter 23: Problems 5, 6, 22, 30, 37 and 38
Chapter 24: Problems 11, 13, 15, 38, 39, 40

1. The mass of the Interstellar Medium of the Galaxy is currently about $5.5 \times 10^{9} \mathrm{M} \odot$. Every year, on average, about $2 \mathrm{M} \odot$ of gas are turned into stars, so we say the star formation rate is $2 \mathrm{M} \odot \mathrm{yr}^{-1}$. (a) At this rate, how long will it take to convert all of the remaining Interstellar Medium into stars, in billions of years? (b) If the age of the Galaxy's disk is 10 billion years, does your answer indicate that we are near the beginning or the end of the disk's star formation lifetime? (c) Some fraction of the mass turned into stars is returned to the Interstellar Medium by winds from stars, Planetary Nebulae, and supernovae. Will this lengthen or shorten the time you found in (a), and why? (6 points)
2. In this problem we'll see how we use observations of the atomic ("HI") gas in the Galaxy to determine how our Galaxy is rotating. Consider the HI gas to be a rotating circular disk with the same gas density everywhere. The gas rotates around the center with the same velocity $v_{\text {rot }}$ at every radius. We use our radio telescope to observe the 21cm spectral line from this gas (see figure on the next page).
(a) If we look in one direction through the Galaxy (known as a Galactic longitude, $l=0^{\circ}$ is the direction of the Galactic Center) as shown in the figure on the next page, explain why we should see a spectrum showing radiation with a range of observed velocities, as shown in the schematic sketch below. Only consider $0^{\circ}<l<90^{\circ}$ for this problem. In particular, why does the observed velocity range go from about $0 \mathrm{~km} \mathrm{~s}^{-1}$ to a well-defined maximum velocity? (3 points)


(b) Write an expression for the "'galactocentric radius" (distance from the center of the Galaxy) of the gas that shows the maximum observed velocity, in terms of $R_{0}$ (the distance from the Sun to the galactic center) and $l$. Call this $R_{1}$. (2 points)
(c) For $l$ relatively small, as in the figure, the Sun's motion is just about perpendicular to the direction we are looking in. So approximate that the Sun's motion is indeed perpendicular. In this case, what is the rotation speed of the gas at galactocentric radius $R_{1}$ in terms of the $v_{\max }$ in the spectrum? (2 points)

By repeating this exercise at other longitudes, we can build up a rotation curve (a plot of rotation velocity vs. radius) for the Galaxy. In general, one has to take into account the Sun's rotation too, because the Sun will have some relative motion compared to the gas we are observing, and, as we saw earlier in the semester, the Earth's orbit around the Sun and its spin.
(d) As a special case, what range of velocities would we observe if we looked along $l=$ $0^{\circ}$ ? (1 point)

Once the rotation curve is determined, then one can turn the problem around and attempt to use the observed velocities of gas clouds to determine their distance from us. These are called kinematic distances. However, there are complications:
(e) At a given longitude in the range $0^{\circ}<l<90^{\circ}$, for observed velocities other than the maximum one, is there a one-to-one mapping of observed velocity with distance from us to the gas? Why or why not? (2 points)

